

Virtual Optimization of the Interaction between Tires and the Vehicle

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Abstract—This article presents a new method to virtually optimize the tire–vehicle interaction in the early vehicle development phase. The basic principle of the method is the definition of characteristic values (CVs) for the tire and the specification of objective maneuver criteria (OMC) describing the driving behavior using a double-track model for the vehicle. The first ones are the design variables, the second ones are used to calculate the fitness/objective function of the optimization. The strategy for their choice is presented and illustrated with examples. After having analyzed this kind of optimization problem, a genetic algorithm proved to be the most suitable, especially if taking into account the presence of many local optima. Its parameter settings and its implementation into the structure of the algorithm are presented. Finally, the method is validated in a case study: a “real” tire (parameterized on a tire test bench) could be reproduced using only its driving behavior as an input. Finally, future prospects and sample applications for this method are presented.

Index Terms—Genetic algorithm, optimization, tire characteristics, vehicle dynamic.

I. INTRODUCTION

From the beginning of the 20th century, when the first cars were constructed for mass production, until the late 70s, the development of vehicles was mainly based on real hardware tests. From then onwards, there was the tendency to shift the development more and more to earlier, virtual design phases, especially in the last few years. This gives the engineers more degrees of freedom. The design variables (DVs) of the vehicle are not fixed in this early design phase so that an optimum can be found more easily. It is also a response to the constantly rising cost and time pressure. Expensive prototypes can partly be replaced by vehicle dynamic simulations, which demand only a fractional amount of the time and costs. Furthermore, it becomes easier for the engineers to understand the physical influences within the complexity of a vehicle. However, real hardware tests are very important and cannot be replaced by simulations, therefore a lot of virtual methods still have to be developed. On the whole, the integrated use of simulation and real hardware tests contribute to faster development times, which make vehicle manufacturers competitive for the world-wide market.

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II. INITIAL SITUATION

The starting point for this investigation has been the analysis of the current tire-vehicle development situation. Until now, the basic, virtual layout of the vehicle dynamics is often based on predecessor tires. The tire development starts in the late vehicle development phase, meaning that the tires can only be adapted to the vehicle which is mostly fixed in its basic characteristics. This is essentially done by test drivers on a test track: they evaluate the driving behavior of different tire specifications. According to their subjective evaluations (SEs), the tire manufacturers modify their specifications to improve the subjective driver’s feeling. This often results in a large number of time and cost consuming loops until the optimum is reached and the tire is approved for production and sale.

Large benefits can therefore be achieved by evaluating and optimizing the tire–vehicle interaction in an earlier phase of the vehicle design process via simulation. With the results of this virtual optimization, the requirement specification for the tires can be written more in detail, allowing the tire manufacturers to develop their tires in a more target-oriented way, saving time and cost. Similar optimization methods, but focusing more on vehicle than tire parameters, were presented e.g. in [1]-[6].

III. CHOICE OF THE OPTIMIZATION PARAMETERS

First of all, the design variables and the objective functions of the optimization problem have to be defined to optimize the tire–vehicle interaction to the driver’s subjective feeling. As the tire has to be optimized, characteristic values (CVs) for the tire have to be found being able to describe its lateral, longitudinal and vertical behavior. In this context, it has to be mentioned that the Magic Formula (MF) tire model has been used, achieving good reproductions of real track tests (on even roads) when focusing only on the lateral and longitudinal vehicle driving behavior (e.g. [7]). To set the CVs, it is necessary to define the basic characteristics of the lateral force and the aligning torque as a function of the slip angle and of the longitudinal force as a function of the slip. Scalar values have been determined to comprehensively describe these characteristics, e.g.

- Off-set and gradient at zero slip/slip angle,
- Coordinates of the peak,
- Coordinates at the limit (maximum slip).

Furthermore, the effects of camber, vertical force and combined lateral-longitudinal forces have to be defined using further CVs. In addition, the vertical stiffness and the relaxation length for the lateral and longitudinal dynamic behavior are set as CVs. The whole set contains 60 CVs.

After having defined the tire CVs, i.e. the design variables, the objective functions for the optimization have to be defined in order to measure the difference between the target driving behavior (based on the driver's feeling) and the actual one. For this purpose, different maneuvers have to be simulated and objective maneuver criteria (OMC) have to be defined. Those are derived from the time signals (e.g. a_y) and in their entirety describe the SEs of the driver.

First of all, a minimal set of maneuvers has to be fixed that covers all important operating conditions of the tire, from static to dynamic and straight running maneuvers. To ensure the linear independence of information of the maneuver set, a correlation analysis (Pearson's and Spearman's index [2]) among all SEs (more than 72 tire evaluations) has been performed, see TABLE I.

TABLE I: CORRELATION (PEARSON) AMONG SEs OF THE 12 MANEUVERS

| | | Maneuvers → | | | | | | | | | | | |
|-------------|----|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Maneuvers ↓ | 1 | | 0.68 | 0.71 | 0.58 | 0.05 | -0.13 | 0.84 | 0.49 | 0.72 | 0.50 | -0.47 | 0.35 |
| | 2 | 0.68 | | 0.83 | 0.34 | 0.13 | 0.09 | 0.83 | 0.51 | 0.79 | 0.54 | -0.39 | 0.43 |
| | 3 | 0.71 | 0.83 | | 0.37 | 0.18 | -0.13 | 0.85 | 0.44 | 0.69 | 0.43 | -0.34 | 0.25 |
| | 4 | 0.58 | 0.34 | 0.37 | | -0.12 | -0.32 | 0.59 | 0.29 | 0.49 | 0.27 | -0.29 | 0.25 |
| | 5 | 0.05 | 0.13 | 0.18 | -0.12 | | 0.74 | -0.01 | 0.01 | -0.22 | -0.07 | 0.26 | -0.34 |
| | 6 | -0.13 | 0.09 | 0.13 | -0.32 | 0.74 | | -0.16 | -0.22 | -0.29 | -0.19 | 0.40 | -0.14 |
| | 7 | 0.84 | 0.83 | 0.85 | 0.59 | -0.01 | -0.16 | | 0.59 | 0.83 | 0.56 | -0.51 | 0.40 |
| | 8 | 0.49 | 0.51 | 0.44 | 0.29 | 0.01 | -0.22 | 0.59 | | 0.75 | 0.79 | -0.28 | -0.17 |
| | 9 | 0.72 | 0.79 | 0.69 | 0.49 | -0.22 | -0.29 | 0.83 | 0.75 | | 0.67 | -0.48 | 0.46 |
| | 10 | 0.50 | 0.54 | 0.43 | 0.27 | -0.07 | -0.19 | 0.56 | 0.79 | 0.67 | | -0.42 | 0.18 |
| | 11 | -0.47 | -0.39 | -0.34 | -0.29 | 0.26 | 0.40 | -0.51 | -0.28 | -0.48 | -0.42 | | -0.19 |
| | 12 | 0.35 | 0.43 | 0.25 | 0.25 | -0.34 | -0.14 | 0.40 | -0.17 | 0.46 | 0.18 | -0.19 | |

Thus, the maneuvers could finally be reduced to five, e.g. the ramp-steer (static) or the single-sine (dynamic, high lateral acceleration) maneuver.

As a next step, OMC have to be found for each maneuver which:

- Are independent from each other (not assignable using other OMC),
- The driver has a high sensitivity on them,
- Are significantly influenced by the tires.

First of all, criteria from literature (e.g. [8]-[11]) and self developed ones have been collected to analyze their influence on the driver (using Sobol's sensitivity analysis [12], e.g. Table II). For that purpose, the tires used for the SEs have been measured on a tire test bench, so that simulation and sensitivity analysis could be performed afterwards. The homogeneity of the given DoE (Design of Experiments) of real tires has been analyzed and it is not as good as an artificial one.

In addition, correlation analysis (Pearson's and Spearman's index) among the OMC have been performed to detect possible dependencies (see e.g. TABLE III). At the end, a minimal set of five OMC has been formulated for each maneuver, see e.g. TABLE IV. Moreover, the correlation between the selected OMC and the SEs has been evaluated to determine the linearity of relationships, e.g. OMC 5 shows a non-linear influence on the driver. Their target values can be set by the user, e.g. by means of Fig. 1 that shows the dependency between the subjective evaluation index and the maximum lateral acceleration of the ramp-steer maneuver (in

this case the number of evaluations has been increased artificially by using an artificial neural network ANN).

TABLE II: SENSITIVITY (SOBOL): OMC-SES (RAMP-STEER)

| | 1st order Sobol | Total Sobol |
|--|-----------------|-------------|
| $\beta/\delta_{St} _{4m/s^2}$ (side-slip angle /steering angle at 4 m/s ²) | 0.58 | 0.58 |
| $\beta/\delta_{St} _{7m/s^2}$ (side-slip angle /steering angle at 7 m/s ²) | 0.14 | 0.13 |
| ... | ... | ... |
| $a_{y,max}$ (maximum lateral acceleration) | 0.02 | 0.02 |
| $\partial\beta/\partial a_y _{4m/s^2,max}$ (side-slip angle gradient at 4 m/s ²) | 0.01 | 0.01 |
| ... | ... | ... |

TABLE III: CORRELATION (PEARSON) AMONG OMC (RAMP-STEER). THE SYMBOLS ARE DESCRIBED IN TABLE II.

| | $a_{y,max}$ | $\beta/\delta_{St} _{4m/s^2}$ | $\beta/\delta_{St} _{7m/s^2}$ | OMC 4 | OMC 5 | $\partial\beta/\partial a_y _{4m/s^2,max}$ |
|--|-------------|-------------------------------|-------------------------------|-------|-------|--|
| $a_{y,max}$ | | 0.28 | 0.41 | 0.04 | 0.59 | 0.12 |
| $\beta/\delta_{St} _{4m/s^2}$ | 0.28 | | 0.88 | 0.54 | 0.77 | 0.49 |
| $\beta/\delta_{St} _{7m/s^2}$ | 0.41 | 0.88 | | 0.61 | 0.89 | 0.61 |
| OMC 4 | 0.04 | 0.54 | 0.61 | | 0.60 | 0.99 |
| OMC 5 | 0.59 | 0.77 | 0.89 | 0.60 | | 0.63 |
| $\partial\beta/\partial a_y _{4m/s^2,max}$ | 0.12 | 0.49 | 0.61 | 0.99 | 0.63 | |

As the last step, the number of design variables (the tire CVs) has to be reduced to the most influencing ones, as this improves the efficiency of the whole optimization. A sensitivity analysis based on Sobol's index identifies the most influencing tire CVs for each maneuver, see e.g. TABLE V for the ramp-steer maneuver. Furthermore, the selected tire CVs should influence the OMC independently, which allows the algorithm a target-oriented optimization. This property can be checked when comparing the Sobol's indices of the single tire CVs in a row. Applying this to the example of the ramp-steer, $a_{y,max}$ can be easily optimized by using $F_{y,max}$ and $F_{y,lim}$. Furthermore, the OMC describing the side-slip angle $\beta/\delta_{St}|_{4m/s^2}$ and $\beta/\delta_{St}|_{4m/s^2,max}$ can also be tuned independently, using K_y for both and additionally $\alpha_{Fy,max}$ for the latter one. As TABLE V only shows an extract of the whole sensitivity analysis, in general there could be found for every OMC sensitive and more or less independent tire CVs.

IV. CHOICE OF THE OPTIMIZATION METHOD

The following objective function has to be minimized by the optimization algorithm, where each OMC is a function of several CVs (lb/ub: lower/upper boundary):

$$\min_{CV \in \mathbb{R}^{n_{DV}}} \text{Err}(\text{OMC}(CV)) \text{ subject to } CV_{lb} \leq CV \leq CV_{ub} \quad (1)$$

For that problem a suitable optimization method has to be chosen that furthermore fits the following requirements:

- The analytical gradients of the OMC with respect to the tire CVs are unknown,
- Due to the complexity of the system, many local optima are present, but the global one has to be found,
- Thanks to computational power the computation time is reduced, so the focus is put on accuracy.

TABLE IV: CORRELATION (PEARSON): OMC-SES (RAMP-STEER). THE SYMBOLS ARE DESCRIBED IN TABLE II.

| | SE |
|--|-------|
| $a_{y,max}$ | 0.55 |
| $\beta/\delta_{Sl} _{4m/s^2}$ | 0.71 |
| $\partial\beta/\partial a_y _{4m/s^2-max}$ | 0.60 |
| OMC 4 | -0.39 |
| OMC 5 | -0.03 |

Several optimization methods have been analyzed (see also [13]), such as deterministic algorithms, Quasi-Monte Carlo methods, evolutionary methods and methods based on global approximations. Finally, a genetic algorithm has been chosen as it shows the best results. Analytical gradients are not necessary and accuracy is high, especially in the presence of many local optima. Also, it does not depend on the quality of an underlying approximation model and it does not take that much time as Quasi-Monte Carlo methods. Genetic algorithms have also been used by e.g. [6] and [14].

The structure of the algorithm has been developed on the basis of the principles of a genetic algorithm [15], see Table VI and Fig. 2. The initial population affects the efficiency of the algorithm, therefore Sobol's sequence (Matlab function `sobolset.m`) with Matousek's scramble (e.g. [16]) has been chosen as it distributes the individuals homogeneously in the design variable space (e.g. [17]).

The structure of the whole optimization process can be summarized in the following main steps:

- 1) A set of tires, characterized via tire CVs, form the initial population (which is the first starting population) of the optimization process.
- 2) To evaluate the fitness of the initial (starting) population, the chosen maneuvers are simulated and their fitness to the target dynamic behavior is judged via OMC. This is done by evaluating the weighted sum (w) of the error between the achieved and the desired variations of the OMC:

$$Err = \sum_i \left| \frac{OMC_i - OMC_{target,i}}{OMC_{max,i} - OMC_{min,i}} \right| w_i \cdot 100 \quad (2)$$

Lower Error (Err) means higher fitness. According to their fitness, only the best tires survive, the rest is disregarded.

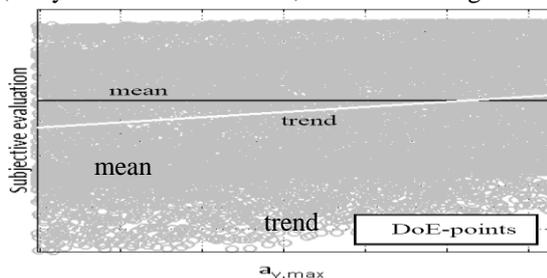


Fig. 1. SE as a function of the maximum lateral acceleration (using ANN, R=0.93)

TABLE V: SENSITIVITY (SOBOL): CVs-OMC (RAMP-STEER). THE SYMBOLS ARE DESCRIBED IN TABLE II.

| | $a_{y,max}$ | $\beta/\delta_{Sl} _{4m/s^2}$ | $\partial\beta/\partial a_y _{4m/s^2-max}$ | OMC 4 | OMC 5 |
|---|-------------|-------------------------------|--|-------|-------|
| K_y (Cornering stiffness) | 0.00 | 0.75 | 0.33 | 0.32 | 0.00 |
| $F_{y,max}$ (max. lateral force) | 0.64 | 0.00 | 0.01 | 0.01 | 0.06 |
| $\alpha_{F_{y,max}}$ (slip angle at max. lateral force) | 0.01 | 0.00 | 0.06 | 0.03 | 0.03 |
| $F_{y,lim}$ (lateral force at limit, i.e. max. slip) | 0.18 | 0.04 | 0.04 | 0.01 | 0.00 |

Thereby the target driving behavior can be set by the user, e.g. using Fig. 1.

- 3) The main step of the genetic algorithm: based on the tires that survived step 2 (parents), the algorithm proposes new tire CVs (children) according to the evolutionary strategy (codification, crossover, mutation, decodification, see Fig. 2).
- 4) The new tire CVs now have to be "translated" to their corresponding MF-parameters so that they can be simulated with the vehicle model. Then the evolutionary process starts again from step 2: the best tires among parents and children are selected according to their fitness, etc.

The following criteria are set to terminate the optimization algorithm:

- Fitness criterion,
- Derivative criterion,
- Time criterion.

Also the parameters of the genetic algorithm determine the accuracy and the speed of the whole optimization. Due to the complexity of the optimization, the parameters could only be set (based on [18]) according to the experience acquired while developing the algorithm. The number of generations is set to 100, which is high enough and normally never reached. As the desired resolution for the error is 1%, it is possible to calculate how many significant digits are necessary to represent the OMC and to estimate the number of required digits for the CVs (in our case at least four). Subsequently, it is necessary to codify them using at least 16 bits, which are sufficient to represent an average of 4.6 significant digits. Furthermore, a decrease function for the population (initial 120, minimum population 70) and the mutation probability raises the efficiency of the algorithm. As an example, the decrease function of the mutation probability p_M (for the starting population is similar) is presented (initial mutation probability $p_{M,0}=0.1$, minimum mutation probability $p_{M,min}=0$, decrease factor $\beta=0.5$ and initial error Err_0).

TABLE VI: BASIC STEPS OF THE GENETIC ALGORITHM

| | |
|-----------------------------------|---|
| Individuals (design solutions) | The tires constitute the population. |
| Gene (design variables) | The tires CVs, the design variables, characterize each tire. |
| Fitness (min. objective function) | The approximation error between desired and actual OMC corresponds to the objective function to be minimized. |

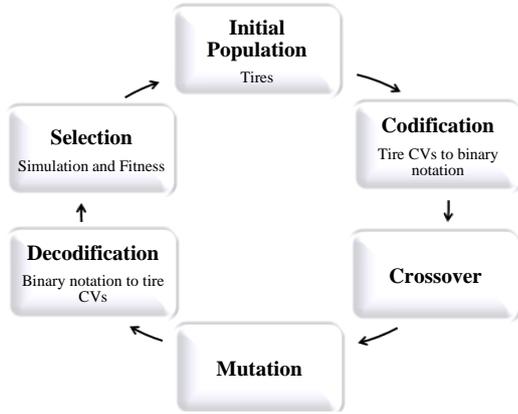


Fig. 2. Cycle of the genetic algorithm

At the beginning, it allows a good coverage of the design variable space, and at the end, when the error gets smaller, a stabilization of the optimization process:

$$p_M(t+1) = (p_{M,0} - p_{M,min}) \left(\frac{Err(t)}{Err_0} \right)^\beta + p_{M,min} \quad (3)$$

Further parameters, such as crossover points and probability, have been set according to Table VII.

V. VALIDATION

In the following case study, the static and dynamic behavior of a real tire has been reproduced using its driving characteristic (the OMC) as input for the optimization algorithm presented in the previous chapters. If the error of the OMC of the resulting, artificial tire is lower than 1%, and the rest of its driving characteristic is matching well the real tire, then the method can be considered as validated.

First of all, the inputs of the optimization, the OMC of the real tire, are extracted from the results of the simulated maneuvers. To reproduce best the overall driving characteristic of the real tire, five maneuvers have been chosen, e.g. the ramp-steer (static) and the single-sine (dynamic, high a_y) maneuver. To describe these maneuvers, more than 20 OMC have been used. According to these input variables the algorithm has to develop an artificial tire whose driving behavior reproduces correctly the original one. The initial population of tires of the genetic algorithm is created according to a DoE using Sobol's sequence with Matousek's scramble.

The results of the optimization are shown in Fig. 3 and Fig. 4. The static, dynamic and straight running behavior of the input (real) tire could be reproduced accurately, each of them having a weighted error lower than 1%.

TABLE VII: PARAMETERS OF THE GENETIC ALGORITHM

| | |
|--|------------------|
| Initial population | $pop_0 = 120$ |
| Decrease factor of the initial population | $\alpha = 0.5$ |
| Minimum number of individuals | $pop_{min} = 70$ |
| Bits for codification | bits = 16 |
| Crossover points | $n_x = 4$ |
| Crossover probability | $p_c = 1$ |
| Initial mutation probability | $p_{m,0} = 0.1$ |
| Decrease factor for the mutation probability | $\beta = 0.5$ |
| Minimum mutation probability | $p_{m,min} = 0$ |
| Max Generations | $n = 100$ |

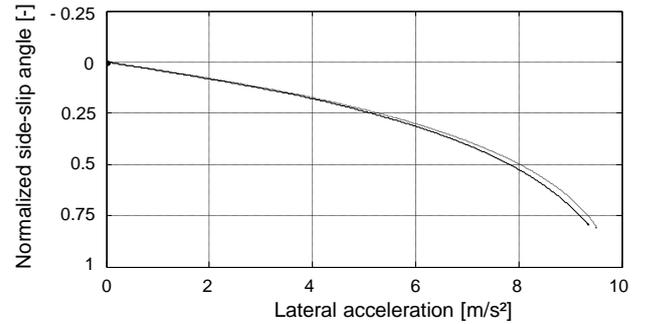
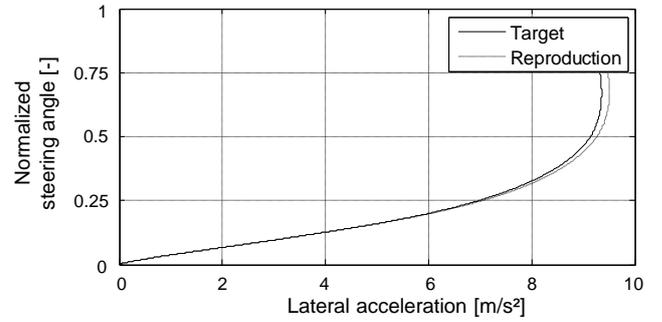


Fig. 3. Static validation – ramp-steer

Moreover, it allows not only to reproduce the OMC with a low error, but also the entire driving characteristic. This is remarkable, as it shows that the CVs, the OMC and the entire optimization algorithm have been chosen correctly.

In general, the optimization takes quite a long time, especially because in every optimization loop the tires have to be simulated with different maneuvers with the vehicle model. Therefore, the idea arose to substitute the simulation with a meta-model, e.g. an ANN. This can reduce the evaluation time, especially when regarding the big amount of optimization loops. The following study shows the optimization only for the ramp-steer maneuver, but the results are transferable to more maneuvers.

The quality of the ANN is reported in TABLE VIII showing the R-values of the cross-validation (see e.g. [19]) for the test set (ramp-steer). There is a lack of quality, especially for the OMC 3 and 5.

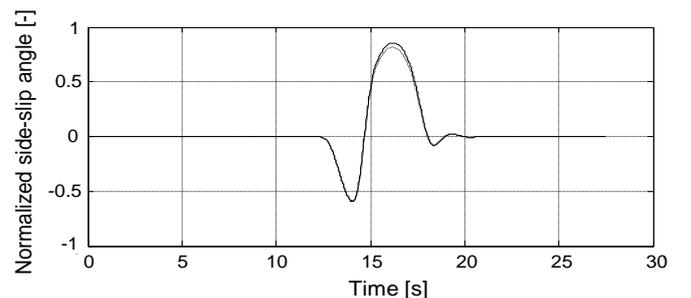
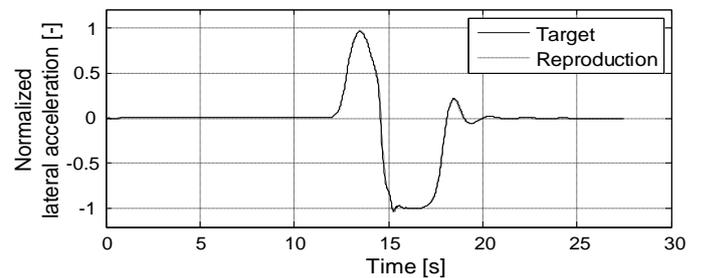


Fig. 4. Dynamic validation – single-sine steer

TABLE VIII: QUALITY OF AN ANN APPROXIMATING THE VEHICLE MODEL

| ANN | R-value |
|-----------|---------|
| for OMC 1 | 0.93 |
| for OMC 2 | 0.99 |
| for OMC 3 | 0.59 |
| for OMC 4 | 0.85 |
| for OMC 5 | 0.66 |

Table IX reports the corresponding results of the optimization with the ANN. At a first view, the ANN seems to give fast and good results, with the fitness and error criterion being lower than the target values. However, when simulating the resulting, artificial tire with the vehicle model and comparing it to its target, significant differences arise. To sum up, if the quality of the ANN is poor it causes a significant approximation error, as expected. Therefore, meta-models should be used only if their quality is high enough.

VI. CONCLUSION

This article presents an optimization method for a simulation based design of the tire-vehicle interaction. A model of the vehicle has to be available, which makes it applicable in the early vehicle development phase when no prototypes are available. The method can be applied in various application fields. First of all, the engineer gets, in the early vehicle development phase, a new degree of freedom for designing the driving dynamics of the vehicle. Furthermore, tires can now be used to compensate variations in the “genes” of the vehicle (mass, wheel base, center of gravity, etc.) or of the suspension (e.g. camber or toe variations) to maintain an unchanged driving behavior. Moreover, the development of tires can now be supported virtually and be more targeted to the subjective feeling of the driver, which reduces time and cost consuming prototype testing loops.

TABLE IX: RESULTS ACHIEVED USING AN ANN

| | ANN | Actual | Target |
|------------------------------|------|--------|----------|
| Cumulated error [%] | 3.06 | 25.38 | - |
| Error criterion [%] | 0.99 | 15.14 | <1.00 |
| Time criterion [generations] | 57 | - | 100 |
| Derivative criterion [-] | 0.10 | - | <0.05 x4 |
| Time [h] (HP Z800, 24 core) | 13.1 | - | - |

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