Patch-Wise Charge Distribution Density for 3D Model Retrieval

Fattah Alizadeh, Alistair Sutherland, and Khaled Moradi

Abstract—The cornerstone of any 3D retrieval system is a shape descriptor. They have a huge influence over the quality of model matching. This paper presents a novel local 3D shape descriptor based on a fact from electrical physics about the distribution of electrical charge on the surface of a solid. After distributing charge \( Q \) on the surface of each model, a number of patches is evenly distributed over the surfaces of each model are extracted and the charge density of each patch is considered as its local descriptor. The bag-of-features framework is leveraged to perform global matching using the local patch descriptors. Experimental results over the SHREC 2011 dataset test the performance of the proposed approach. Beside the robustness of the proposed approach against linear transformation and noise, its retrieval quality is quite comparable to the state-of-the-art approaches participated in the SHREC 2011 contest.

Index Terms—3D model retrieval, shape descriptor, electrical charge density, bag of feature.

I. INTRODUCTION

Content-Based Information Retrieval (CBIR) is among the interesting research fields which have gained enormous attention during recent decades. One of the recent and most challenging domains of CBIR is 3D model retrieval which tries to describe, match and retrieve similar 3D models to a desired query. Due to the recent proliferation in 3D modelling tools such as 3D scanners, cameras and modelling software, the number of available 3D models in publicly available repositories over the internet is growing. Accordingly, describing models in an efficient and effective manner is of great value to the relevant researchers.

The main step and foundation of any typical 3D retrieving systems is to describe the query and other available models in a useful and discriminative way. Thus, the major focus in this field is to find a descriptor by which all of the characteristics of the models can be represented. Many techniques have been proposed to represent the models using a numerical feature vector or an appropriate graph [1]-[3].

Beside the higher retrieval quality of any shape descriptor, it should meet some other critical requirements viz. robustness to linear transformations (no need for pose normalization), invariance to noise, simplification and deformation and finally fast retrieval ability. Although some of the available shape descriptors offer good ability in terms of time and accuracy, it is still an open question to propose a shape descriptor which meets all of these requirements.

In this paper, in order to describe 3D models via local descriptors, we borrow a fact from electrical-physics about the tendency of electrical charges to accumulate at sharp convex areas and to diminish at sharp concavities on the surface of a solid. After calculation of the charge density on the arbitrary surface of a solid model, we define some patches centred on some random feature points uniformly sampled on the surface. The amount of accumulated charge per area for each patch is employed as a rich local descriptor and the bag-of-features (BoF) approach is leveraged to perform global shape matching using the local patch descriptors. As will be discussed latter, the proposed descriptor is robust to linear transformation as well as to a variety of surface modifications. Fig. 1 shows the distribution of electrical charge on the surface of some 3D models.

The idea of charge density distribution has been recently utilized to retrieve 3D models [4]. But, instead of using local descriptors; the authors directly employed the scalar value of charge density for each triangular face in order to describe 3D models. Assigning one scalar value to each face needs more space to store the descriptor information and will affect the retrieval speed. Later in this paper, we compare the ability of the proposed approach with this similar approach in both terms of speed and accuracy.

The outline of the paper is as follows: in Section II we survey related works on the bag-of-features approaches in 3D model retrieval. The proposed shape descriptor is mainly described in the Section III. Our experimental set up is presented in Section IV, whereas concluding remarks and future extensions of our work will be discussed in Section V.

II. RELATED WORKS

Recently, shape descriptors have been extensively studied in the context of 3D shape retrieval. Thus, a number of approaches are available now by which 3D models can be described in a discriminative manner. For precise details about these approaches we refer the readers to the survey paper by Tangelder and Veltkamp [5]. Since our proposed approach mainly uses the Bag of Features framework, this
Section is dedicated to give a brief background on BoF in the 3D Retrieval domain.

A typical content-based image retrieval system using the BoF framework has 3 major steps: 1): feature point selection, 2): visual dictionary building and 3): histogram generation. Fig. 2 illustrates the simple BoF framework for 3 different objects. Thanks to the BoF framework, the local 3D shape descriptors can be utilized for global shape matching. Several works have described models using the BoF framework.

The Bag-of-Features (Words) was originally devised for use in the text-based information retrieval systems. In the BoF framework, a text (such as document or sentence) is considered as an unordered collection of words disregarding grammar or even word order. Despite the simplicity of such a representation, the retrieval methods that use the BoF framework often have shown a high retrieval performance, so a great deal of research has been conducted to employ BoF in 2D and 3D Image Retrieval [2].

Tabia et al., leveraged BoF for describing models using some closed curves associated to feature points located at the extremities of 3D models [6]. In their work, the belief function was used to calculate the dissimilarity measure between models.

Fig. 2. A typical BoF framework in a 3DOR system.

The spin image [7] is used as a local descriptor along with the BoF framework in several works [8], [9]. In the work of Shan et al., a probabilistic framework was introduced in order to support partial matching [9]. They used the spin image as a descriptor and proposed the “Shapeme” histogram projection algorithm which can match partial objects. The spin image has been extended later by Darom and Keller to offer the Scale-Invariant form of Spin Image (SISI) [8]. The SISI is obtained by applying the spin image over the estimated local scale of feature points on the surface of models.

A multi-level BoF is proposed by Toldo et al., [10]. The Shape Index descriptor (SI) is used to do the segmentation process. And then four different shape descriptors namely Shape Index, Radial Geodesic Distance, Normal Direction and Geodesic context are applied on each segment. Based on the type of descriptor and the different number of bins in the BoF framework, several histograms are built to describe the models for partial matching purpose.

The BoF along with depth-buffer images are used in the work of Lian et al., [1]. The authors used the canonical form of models to extract the depth-buffer views. The SIFT descriptor [11] is utilized to locate and describe salient points on each view and finally the BoF framework is leveraged for model histogram building and comparison. The main contribution of their work is the new matching scheme which they called Clock Matching. Their method achieved very good rank among the participants of the SHREC’11 contest [12].

The main obstacle that may reduce the effectiveness of the BoF framework in 3D model retrieval domain is the loss of valuable information; because all of the spatial relationships among the feature points are discarded, their descriptive capability is severely constrained.

Behmo et al., utilized the commute graph to overcome this problem [3]. After detecting the feature points, the authors connected the features which are spatially close together. Then the features whose descriptors are quantized to the same index in the visual vocabulary are grouped together to collapse the graph. The commute time between the features nodes are considered as the spatial information among the features to enhance the retrieval quality of BoF.

Recently in [2], an interesting method is proposed to consider the spatial relationship among the features. Instead of using a histogram for each feature, images are described as histograms of pairs of features and the spatial relations between them (visual expressions). The Multi-Scale Diffusion Heat Kernel is employed as a shape descriptor. The same “visual expression” technique is leveraged by Lavoué very recently [13]. Firstly, some patches are defined associated with feature points uniformly sampled on the surface. Then, each patch is described by projecting the geometry onto the eigenvectors of the Laplace–Beltrami operator. A BoF framework augmented with the visual expression technique is employed to support partial matching in a spatial-sensitive manner.

Very recently Tang and Godil offered an evaluation for local shape descriptors using the BoF scheme [14]. They systematically evaluated several local shape descriptors for different feature point selection criteria. Some effective parameters for BoF such as dictionary size and sample point count are tested under different shape descriptors such as Main Curvature, Gaussian Curvature, Shape Index and Normal Distribution.

Generally speaking, apart from discarding the spatial relationship among features, the BoF framework can be considered as one of the most useful and popular approaches in 3D retrieval domain.

III. PROPOSED APPROACH

In this Section we firstly give a brief introduction to calculation of distributed charge density using the Finite Element Method and then our proposed algorithm is discussed in detail.
A. Calculation of Charge Density Distribution on the Arbitrary Surface

When a pre-defined electrical charge \( Q \) is placed on the surface of an arbitrary surface, it is spread over the surface so that its distribution follows a well-known fact in physics of electricity which says: "the electric charges on the surface of a conductor tend to accumulate at the sharp convex areas and diminish at the sharp concave areas."

Inspired by this fact, we try to use the density of distributed charge over the surface of models as a local shape descriptor. To this end, 3D models are treated as conductors which are placed in a free space (a space with no any electric charge) and the electrical charge is distributed on the surface of them. Since the 3D models have arbitrary surfaces, it is not possible to calculate the charge density on the surface using an analytical approach. Thus, a Finite-Element-Method (FEM) is utilized to this end. We used the approach proposed by Wu and Levine to calculate the charge density [15]. Their approach can be briefly expressed as follows:

Firstly, the reciprocal electrical potential of every pairs of faces are identified by Equation 1:

\[
\varphi(r) = \frac{q}{4\pi\varepsilon_0 |r - r'|}
\]

where \( q \) is the position of free space and \( r \) is the vector position of observation point and \( r' \) is the position of charge \( q \).

Since, all of triangular faces contribute in the potential \( \varphi(r) \), it can be re-written as follows:

\[
\varphi(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{|r - r'|} ds' = \varphi_1 + \varphi_2
\]

where \( S \) is the total surface area, \( \rho(r') \) is the charge density at \( r' \) and \( s' \) is the area over \( S \). In order to calculate \( \varphi(r') \) using FEM, the model surface \( S \) is considered as \( N_T \) triangles \( T_1, T_2, \ldots, T_{N_T} \). Therefore, the Equation 3 can be expressed as:

\[
\varphi(r) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N_T} \left( \rho_i \int \frac{1}{|r - r'|} ds' ight), i = 1, 2, \ldots, N_T
\]

On the other hand, the total charge \( Q \) is equal to the sum of charges on each triangle. Thus:

\[
Q = \sum_{i=1}^{N_T} \rho_i s_i
\]

Here \( \rho_i \) and \( s_i \) are charge density and surface area of triangle \( k \) respectively. Using Equations (3) and (4), a set of linear equations with \( N_T+1 \) unknowns \( \rho_1, \rho_2, \ldots, \rho_{N_T} \) and \( \varphi(r) \) in the form of \( A \times \rho = Q \) are obtained which can be easily solved using a linear equation solver. More details on the above equations can be found in [15].

B. Patch-Wise Charge Density Descriptor (Patch-Wise CDD)

Fig. 3 depicts the main steps of our approach; it starts with density calculation of distributed charge \( Q \) on each triangular face of models. Then, \( N_p \) uniformly distributed patches are created on the surface of each model for dividing models into \( N_p \) sub-regions. The electrical charge density for each patch is utilized as the local descriptor and is coupled with the BoF framework to describe entire models surface.

C. Uniformly Distributed Patch Creation

Aiming for dividing the 3D model into evenly distributed patches we employed a uniform sampling algorithm of the feature points proposed by Guillaume [13]; \( N_p \) surface points are randomly selected on the model surface as an initial set of seeds to feed Lloyd’s relaxation algorithm. Until a desired convergence, the recently located seeds are used to find the new seeds by iteratively moving the seeds to the centroids of their Voronoi cells. As shown in Fig. 4. (a) the final seeds, which are considered as the feature points, are uniformly distributed after 40 iteration of Lloyd’s algorithm.

![Fig. 3. The proposed retrieval framework.](image)

In order to divide the model surface into patches, an equal-radius geodesic circle is assigned to each feature point \( v \) by considering a set of connected triangles belong to the geodesic circle of centre \( v \) and radius \( r \). In order to achieve an overlap region and to avoid large gaps between adjacent patches we define the radius \( r \) using Equation (5):

\[
r = \frac{d}{2} + 0.12 \times d
\]

where \( d \) is the average geodesic distance between each pairs of adjacent feature points. The \( 0.12 \times d \) was inserted to \( r \) to achieve overlap between patches. Fig. 4. (b) depicts three feature points and their relevant geodesic circular patches.

![Fig. 4. (a): 250 uniformly distributed seeds after 40 Lloyd’s iterations. (b): Three extracted patches for sample feature points.](image)

D. Patch Descriptors

The scalar value of electrical charge density for each face is used as a local descriptor to describe the circular patches. The total amount of distributed charge on the entire surface of patch \( p \) can be easily found and considered as the patch density which acts as local descriptor.

The following equation is used to define electrical charge density per patch \( p \) which comprises \( N_p \) faces:

\[
\text{Charge Density}_p = \sum_{i=1}^{N_p} \rho_i
\]
part to describe the part and leverage it to support partial matching. We will test this ability of the CDD shape descriptor in our next work.

IV. EXPERIMENTAL RESULTS

We tested the proposed local descriptor on the models in the standard dataset SHREC 2011 which contains 600 watertight models equally categorized into 30 classes.

Since the charge density descriptor is robust against simplification [4], all of the models are down-sampled into 3000 triangular faces before any computation to speed up the retrieval process. During the entire process we have chosen \( r = 40 \) for the numbers of iterations in the Lloyd’s relaxation algorithm.

In the sequel, we examine the ability of the proposed approach on noisy models as well as the effect of the BoF parameters. Additionally, we experimentally evaluate our algorithm using Precision-Recall curves over the mentioned dataset.

A. Robustness against Noise

Some noisy models were created with the aim of examining the effect of noise. To this end, we created noisy models by randomized displacement of the vertex coordinates determined by noise level \( nl \) (\( nl \) is the ratio of largest displacement to the longest edge of the object’s bounding box), and calculated the charge density. The proposed system, as depicted in Table I, has shown good ability in retrieving similar models to the noisy Teddy and Hand models with 0.2 of noise level. It should be mentioned that for some models such as Lamp and Pliers the results were perfect so that all of 10 retrieved models were correct. On the other hand, for some queries such as models from classes Man and Bird1, the results include more incorrectly retrieved models. This can be because of availability of similar models in different classes (e.g. Man and Woman classes or Bird1 and Bird2 classes).

<table>
<thead>
<tr>
<th>Noisy Query</th>
<th>Hand-1</th>
<th>Hand-2</th>
<th>Hand-3</th>
<th>Hand-4</th>
<th>Lamp-1</th>
<th>Lamp-2</th>
<th>Teddy-1</th>
<th>Teddy-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Level</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

B. The BoF Framework Parameters

After testing the effect of different parameters of the algorithms (namely the dictionary size and the number of feature points for the BoF framework), and observing their effect on evaluation factors, we found that \( \text{Dic}_\text{size}=50 \) and \( N_{\text{fp}}=400 \) are the best values for the dictionary size and the feature point count, respectively. As shown in Table II, increasing dictionary size leads to improving Discounted Accumulative Gain factor (DCG), but since there are no meaningful differences between 50, 200 and 400, we have selected the smaller number as dictionary size. The smaller size of the dictionary will result in lower number of comparisons and a higher speed of the retrieval process.
TABLE II: EVALUATION FACTORS FOR DIFFERENT DICTIONARY SIZES IN THE BOF FRAMEWORK (400 SURFACE FEATURE POINTS)

<table>
<thead>
<tr>
<th>Dic. Size</th>
<th>NN</th>
<th>FT</th>
<th>ST</th>
<th>E</th>
<th>DCG</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.5712</td>
<td>0.3810</td>
<td>0.5134</td>
<td>0.3805</td>
<td>0.6972</td>
</tr>
<tr>
<td>10</td>
<td>0.6501</td>
<td>0.4092</td>
<td>0.5611</td>
<td>0.4438</td>
<td>0.7271</td>
</tr>
<tr>
<td>20</td>
<td>0.7014</td>
<td>0.4360</td>
<td>0.5817</td>
<td>0.4668</td>
<td>0.7612</td>
</tr>
<tr>
<td>50</td>
<td>0.7846</td>
<td>0.5078</td>
<td>0.5862</td>
<td>0.5137</td>
<td>0.8004</td>
</tr>
<tr>
<td>200</td>
<td>0.7939</td>
<td>0.5142</td>
<td>0.5780</td>
<td>0.5253</td>
<td>0.8035</td>
</tr>
<tr>
<td>400</td>
<td>0.8135</td>
<td>0.5220</td>
<td>0.5942</td>
<td>0.5288</td>
<td>0.8112</td>
</tr>
</tbody>
</table>

C. Time Complexity

We implemented our algorithm in MATLAB and some of its time-consuming parts were written in C++. Before any computation, all of the models are down-sampled into 3000 triangular faces. The total process for simplification, charge density calculation and patch creation for each model takes an average of 9.3 seconds. And the average time for search a query on SHREC’11 dataset and retrieve models on a 3GHz PC with 4GB RAM is 11.4 seconds.

D. Comparing with Other Approaches

To verify the effectiveness of the proposed approach we applied our algorithm to the models in the standard SHREC’11 dataset and compared the results to the state-of-the-art BoF-based approaches participated in the SHREC’11 contest. It should be noted that 6 among 9 participants of SHREC’11 contest have utilized the BoF framework which confirms the ability and popularity of BoF in 3D retrieval domain. Additionally, we compared our approach with the similar approach of BoF_CDD [4]. To see more descriptions about these approaches we refer the readers to the Section 2 of the current paper and the report paper published on the SHREC’11 contest [12].

The standard precision-recall curve was employed in order to compare the methods, where the Precision and Recall factors are defined as follows:

\[
\text{Precision} = \frac{X}{A}, \quad \text{Recall} = \frac{X}{B}
\]

Here \(X\) is the number of relevant models retrieved, \(A\) is the number of retrieved models and \(B\) is the number of relevant models in the target dataset. An ideal Precision-Recall curve has precision equal to 1 for all values of recall (all relevant models are retrieved before any irrelevant ones).

![Precision-Recall Curve](image)

**Fig. 5.** The precision-recall curve of the BoF-based approaches participated in the SHREC 2011 contest.

As illustrated in Fig. 5, our approach outperforms most of the contestants of SHREC’11 and is quite comparable to the best approaches; FoG and MDS_CMsBoF. The MDS_CM_BoS descriptor is still the best one which is, as stated before, because of their interesting matching approach, Clock-Matching, that compares all possible pairs of relevant views to find the similarity measurement.

Comparing the discriminative ability of the proposed approach to that of BoF_CDD in Fig. 5 illustrates that the results of both approaches are almost similar. But in term of retrieval speed, the proposed approach is slightly faster than the BoF_CDD in searching a query on SHREC’11 dataset; the average time for search a query and retrieve similar models for BoF_CDD takes 13.3 seconds while it is 11.4 seconds for the proposed algorithm. A reason for this can be the smaller number of feature points (400 points) we use in comparison to 1000 feature points that the BoF_CDD uses during entire process of retrieval system.

V. CONCLUSIONS AND FUTURE WORK

In this paper we utilized a fact from electrical physics to describe 3D models; a set of local patches associated with some uniformly distributed feature points on the surface of models were identified. The charge density of each patch was used as a local descriptor coupled with the BoF framework to perform global matching of the models. Several experiments were conducted over the models in the SHREC’11 dataset to evaluate the retrieval ability of the proposed approach. Experimental results support the theoretical claims that the proposed shape descriptor is robust against noise and deformation.

Augmenting our approach so that it can support partial matching is one the objectives of the current research. Therefore, in the future we will try to extend it so that the similarity measurement defined based on the sub-part resemblance. Furthermore, like other standard BoF-Based approaches, the proposed approach suffers from disregarding the relationship between feature points. So the other improvement can be solving this issue by considering the spatial relation among the extracted feature points.

REFERENCES


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