Constraint Programming Based Production Planning in a Petrochemical Industry

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Abstract—**In this article, we provide a constraint programming based optimization model to determine the optimal production planning to maximize profit in a petrochemical industry. The proposed model despite its simplicity overcomes the limitations of the mixed integer linear programming model available in literature. The benefits of the proposed model are demonstrated on various instances of two case studies and it is shown that the proposed formulation enables an increase in the profit in the range of 0.58 to 3.68%.**

Index Terms—**Combinatorial optimization, constraint programming, petrochemical industry, production planning.**

I. INTRODUCTION

A petrochemical industry uses series of complex networks to convert feedstock such as oil and gas to produce primary petrochemicals such as methanol, ethylene, propylene, benzene, toluene, xylene, etc. These primary petrochemicals are subsequently converted into petrochemical intermediates and derivatives which are ultimately transformed into products used in the market. The petrochemical industry operates at very high production levels which is achieved either through the large scale of the individual equipment or through the large scale of the entire plant [1]. A variety of optimization models have been proposed for efficiently operating the petrochemical plants. These include efficient production planning [2], mergers and acquisitions [1], integration of refineries and petrochemical plants [3], capacity expansion [4], efficient spatial organization of petrochemical plants [5], efficient job scheduling [6], and optimal supply chain management [7].

In this article, we use Constraint Programming (CP), an optimization technique that has its origins in the Artificial Intelligence and Computer Science Community [8], for solving the production planning problem in an industry. A Mixed Integer Linear Programming (MILP) model has been proposed in literature [2] to determine efficient production plans to guide the petrochemical industry development in Saudi Arabia. However, this formulation restricts the productions to 'artificial' levels thereby leading to suboptimal profit. In this article, we propose a CP based optimization formulation which overcomes the drawbacks of the MILP formulation in literature. The benefit of the proposed formulation has been demonstrated on eight different instances involving two distinct requirements. In the next

section, we provide a brief description of CP and this is followed by the description of the problem statement. In the next section, we review the MILP formulation in literature and describe its limitations. In the subsequent section, we present a CP based optimization formulation to solve the production planning problem and subsequently demonstrate the applicability of this formulation on various instances of two scenarios. We conclude the article by discussing the developments in this article and present possible future work in this direction.

II. CONSTRAINT PROGRAMMING

CP was predominantly used to solve Constraint Satisfaction Problems (CSP) and has been extended to solve single [9], [10] and multi objective optimization problems [11]. CP has found widespread application in solving combinatorial optimization [12]-[14] such as Assignment Problems [15], Network Problems [16], [17], Production Planning/Scheduling, Personnel Allocation. The benefits of CP include the ease of modeling as it does not restrict the user like mathematical programming, guarantee of the optimality of the solution even for non linear discrete problems and the ease of determination of value added solutions such as realizations and K-best solutions. Moreover, unlike other AI based optimization algorithms such as Genetic Algorithms, Simulated Annealing, Teaching Learning Based Optimization, it does not require any set of parameters that need to be tuned.

CP is an implicit, tree-based enumeration technique that intelligently uses the propagation of constraints to reduce the domain of the variables. Unlike optimization problems, the objective in CSP is to determine a feasible solution which is a set of singleton values for all the decision variables from its individual domains, while satisfying all the given constraints. In the initial phase, CP uses the constraints to eliminate as many infeasible combinations as possible by propagating the constraints. If it happens that a feasible solution is discovered in this phase, the procedure is terminated. However, it is very rare that this phase would lead to a feasible solution. Under such circumstances, a non deterministic choice is made on any of the decision variables i.e., a decision variable is fixed to one of the possible values from its domain. This procedure is referred to as choice point in CP literature. Subsequently, the constraints are propagated to reduce the domain of the other variables based on this choice point. If all the variables become singleton, the choice point is said to have led to a 'success' as a feasible solution is achieved. However, if the choice point leads to a situation where in the domain of at least one of the variables becomes empty, it is said to have led to a 'failure' and hence a revision of the choice point is

necessary. It can be noted that at times, the selection of a single choice point is not sufficient and in such cases, a series of choice points are made till a failure or success is achieved. In case of failure, the latest choice point can be revised. Thus the selection of choice point and constraint propagation is carried repeatedly till a feasible solution is obtained.

CP solves optimization problems by transforming it into a constraint satisfaction problem by converting the objective function into a constraint with an appropriate bound. With the discovery of every feasible solution, the bound of the objective constraint is revised so as to determine an improved solution. This process is continued till the search space is completely explored or the problem becomes infeasible. The last updated bound corresponds to the globally optimal solution of the optimization problem. A detailed explanation on various aspects can be obtained from literature [8].

The drawbacks of CP include lack of use of relaxations and other geometrical properties of the problem to guide its search to the optimal solution. This at times makes CP computationally expensive for certain problems as it very often requires considerable computational resources to confirm the optimality of the solution. Many of the software restrict the use of CP to discrete variables and enforce various limitations on the use of continuous variables. Recent trends to overcome these drawbacks include the development of hybrid algorithms [18-20] which harness the complimentary benefits of CP and mathematical programming techniques.
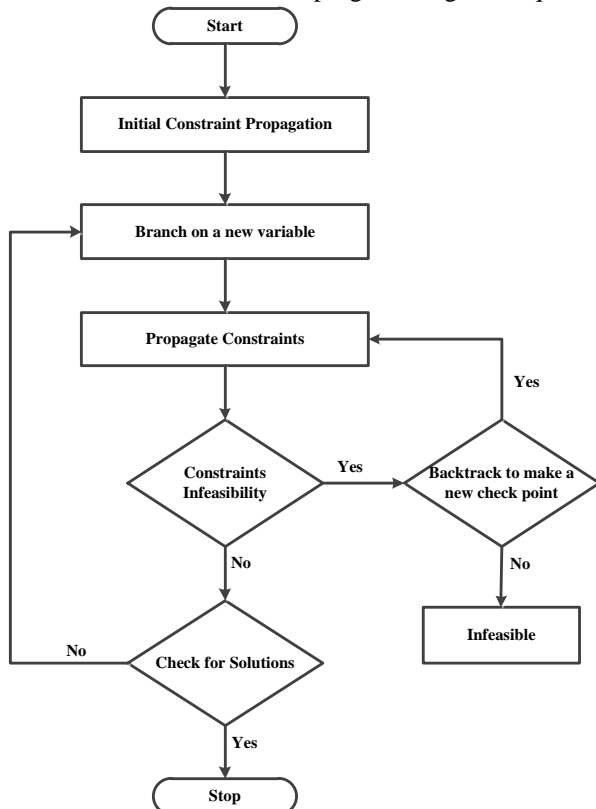

Fig. 1. Schematic of CP algorithm.

III. PROBLEM DESCRIPTION

The production planning problem is combinatorial in nature and involves deciding the quantities of I products that need to be produced from a given set of J processes so as to maximize the profit. Each of the process is assumed to produce only one product while some of the processes can produce the same product by using a specified amount of raw material. Each of these processes can be operated at low, medium or high level based on the amount of production. The production and investment cost of each of the processes depends on the amount of product (thus the production levels) that is produced form a particular process. The petrochemical industry has limited investment budget and has limited supply of raw materials thereby restricting the amount of products that can be produced. The production cost and investment cost are usually modeled as piecewise linear functions between the production levels as shown in Fig. 1.
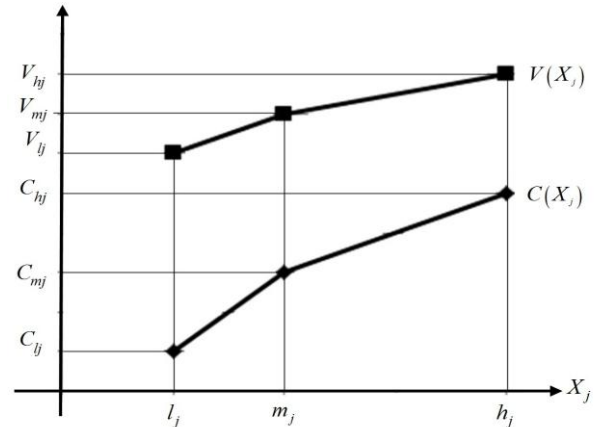

Fig. 2. Production and Investment cost varies nonlinearly with respect to capacity of production levels.

In addition to the constraints discussed above, sometimes a constraint is enforced that no product should be produced by more than one process. This requirement is known as the 'unique process requirement'. Table I and Table II shows an example of one such problem wherein 54 processes are available which can be operated at three different levels and can produce a maximum of 24 products. Without loss of generality, we have used this data for demonstrating the results of this work. In Table I, N_T denotes the name of the product whereas N_S indicates the name of the process. The notations l_j, m_j and h_j represent the units of production of the corresponding products in the low level, medium level and high level production capacity. The amount of raw material R_1 and R_2 required by the processes for producing per unit of the product is given by b_{j1} and b_{j2}. The per unit sale price of the product is given by E_j. As can be seen from Table I, Product F can be produced from six different processes (Process 10 to Process 15) and each of these processes have a different raw material requirement, production cost and investment cost that vary with the three different levels of production. If the unique processes requirement is not included, this product may be produced by more than one process of the six possible processes whereas if the unique process requirement is included, the Product F is to be produced by only one of the six processes. It should be obvious that the inclusion of this constraint can lead to the reduction in profit. Some of the products in the Table I such as Process $19-24$, have zero requirement of raw material R_1 and R_2. It should be noted that these products use some other raw materials which do not have an upper limit in the availability.

If upper limits on these raw materials exist, they too would have to be accommodated in the model formulation.

h_j which leads to suboptimal solutions as an 'artificial' limit is implemented on a highly profitable product.

TABLE I: PRODUCTION LEVELS OF PROCESSES

N_T	N_S	b_{j1}	b_{j2}	E_j	l_j	m_j	h_j
A	1	0	3	3	35	44	91
	2	0	2	5	38	55	93
	3	0	2	6	13	69	84
B	4	0	3	5	38	51	73
	5	0	2	7	29	58	77
C	6	0	5	6	13	47	98
	7	0	1	5	18	63	75
D	8	0	1	3	26	48	95
E	9	0	1	6	39	56	87
F	10	0	4	2	39	61	10
	11	0	3	4	14	67	73
	12	0	4	2	40	69	84
	13	0	1	4	39	57	74
	14	0	5	7	25	45	99
	15	0	4	2	34	45	71
G	16	0	2	7	14	48	94
	17	0	4	5	23	66	95
H	18	0	1	7	38	48	97
I	19	0	0	5	34	65	73
	20	0	0	4	39	48	82
J	21	0	0	2	30	68	78
	22	0	0	6	11	51	95
K	23	0	0	5	36	46	83
	24	0	0	5	38	48	98
	25	1	0	2	31	59	76
L	26	1	0	4	33	55	78
M	27	1	0	3	33	51	75
N	28	1	0	4	22	65	75
	29	4	0	2	30	58	97
	30	1	0	5	15	57	88
O	31	5	0	3	31	68	87
	32	4	0	5	10	49	75
	33	4	0	6	18	63	96
P	34	3	0	4	11	63	89
Q	35	0	0	3	13	52	81
	36	0	0	7	35	58	86
	37	0	0	4	31	43	83
R	38	0	0	6	19	42	73
	39	0	0	2	39	56	78
	40	0	0	2	11	64	74
S	41	0	0	5	23	69	76
	42	0	0	7	21	44	78
T	43	0	0	2	33	58	83
	44	0	0	4	34	55	72
	45	0	0	6	15	41	98
U	46	4	0	7	25	51	99
	47	3	0	3	23	45	85
	48	3	0	7	30	64	85
	49	2	0	4	31	50	81
V	50	0	0	7	33	56	98
	51	0	0	2	18	45	82
W	52	0	0	2	31	59	74
X	53	0	0	4	30	48	94
	54	0	0	2	15	60	82

TABLE II: PRODUCTION AND INVESTMENT COSTS

N_T	N_S	Cl_j	Cm_j	Ch_j	Vl_j	Vm_j	Vh_j
A	1	85	103	244	135	162	350
	2	91	125	239	144	195	347
	3	41	153	198	69	237	297
B	4	91	117	183	144	183	271
	5	73	131	188	117	204	280
C	6	41	109	262	69	171	375
	7	51	141	177	84	219	267
D	8	67	111	252	108	174	362
E	9	93	127	220	147	198	322
F	10	93	137	254	147	213	369
	11	43	149	167	72	231	255
	12	95	153	198	150	237	297
	13	93	129	180	147	201	269
	14	65	105	267	105	165	381
	15	83	105	183	132	165	269
G	16	43	111	249	72	174	358
	17	61	147	234	99	228	344
H	18	91	111	258	144	174	370
I	19	83	145	169	132	225	257
	20	93	111	213	147	174	310
J	21	75	151	181	120	234	274
	22	37	117	249	63	183	359
K	23	87	107	218	138	168	316
	24	91	111	261	144	174	374
	25	77	133	184	123	207	275
L	26	81	125	194	129	195	287
M	27	81	117	189	129	183	279
N	28	59	145	175	96	225	265
	29	75	131	248	120	204	360
	30	45	129	222	75	201	325
O	31	77	151	208	123	234	310
	32	35	113	191	60	177	281
	33	51	141	240	84	219	351
P	34	37	141	219	63	219	323
Q	35	41	119	206	69	186	302
	36	85	131	215	135	204	316
	37	77	101	221	123	159	319
R	38	53	99	192	87	156	280
	39	93	127	193	147	198	286
	40	37	143	173	63	222	262
S	41	61	153	174	99	237	265
	42	57	103	205	93	162	298
T	43	81	131	206	129	204	304
	44	83	125	176	132	195	263
	45	45	97	268	75	153	381
U	46	65	117	261	105	183	375
	47	61	105	225	99	165	325
	48	75	143	206	120	222	306
	49	77	115	208	123	180	304
V	50	81	127	253	129	198	366
	51	51	105	216	84	165	313
W	52	77	133	178	123	207	267
X	53	75	111	249	120	174	358
	54	45	135	201	75	210	298

This problem has been previously modeled as a MILP but this formulation enforces that either the production from a process is between l_j and m_j or between m_j and h_j thereby preventing production using multiple levels. This MILP formulation also restricts the production of a product to

IV. CONSTRAINT PROGRAMMING FORMULATION

In this section, we propose a CP formulation for the production planning problem described in the previous section. As can be seen in Fig. 2, there is a straight line

governing the production cost between l_j and m_j and another straight line between m_j and h_j. Each of these two lines can be represented in the slope-intercept form. In the formulation below, $PCmhM_j$ and $PCmhC_j$ denote the values of slope and intercept respectively for the line connecting m_j and h_j whereas $PClmM_j$ and $PClmC_j$ denote the values of slope and intercept respectively for the line connecting l_j and m_j. Similar to the lines for production cost, there are two other lines governing the investment cost between the intervals of (l_j, m_j) and (m_j, h_j) for the jth process. In the formulation below, $IClmM_j$ and $IClmC_j$ will denote the value of slope and intercept respectively for the investment cost line connecting l_j and m_j whereas $ICmhM_j$ and $ICmhC_j$ will denote the value of the slope and intercept respectively for the investment cost line connecting m_j and h_j. The parameter P_{ij} is a binary parameter with a value of 1 if the ith product is produced from the jth process.

$$\text{Maximize:} \qquad \text{Profit} = \sum_{j=1}^{J} E_j X_j - PC_j \qquad (1)$$

$$s.t \quad zlm_j = \min\left(1, xlm_j\right) \qquad \forall j = 1,2,...,J \qquad (2)$$

$$zmh_j = \min\left(1, xmh_j\right) \qquad \forall j = 1,2,...,J \qquad (3)$$

$$Xlm_j = 0 \| \left(Xlm_j \geq l_j \,\&\, Xlm_j \leq m_j\right) \quad \forall j = 1,2,...,J \quad (4)$$

$$Xmh_j = 0 \| \left(Xmh_j \geq m_j \,\&\, Xmh_j \leq h_j\right) \forall j = 1,2,...,J \quad (5)$$

$$PCmh_j = \left(PCmhM_j\right)\left(Xmh_j\right) + \left(zmh_j\right)\left(PCmhC_j\right) \qquad (6)$$

$$ICmh_j = \left(ICmhM_j\right)\left(Xmh_j\right) + \left(zmh_j\right)\left(ICmhC_j\right) \qquad (7)$$

$$PClm_j = \left(PClmM_j\right)\left(Xlm_j\right) + \left(zlm_j\right)\left(PClmC_j\right) \qquad (8)$$

$$IClm_j = \left(IClmM_j\right)\left(Xlm_j\right) + \left(zlm_j\right)\left(IClmC_j\right) \qquad (9)$$

$$X_j = Xmh_j + Xmh_j \qquad (10)$$

$$PC_j = PClm_j + PCmh_j \qquad (11)$$

$$IC_j = IClm_j + ICmh_j \qquad (12)$$

$$\sum_{j=1}^{J} b_{jt} X_j \leq R_t, \forall t = 1,2,...,T \qquad (13)$$

$$\sum_{j=1}^{J} IC_j \leq B \qquad (14)$$

$$\sum_{j=1}^{J} P_{ij} X_j \leq D_i \quad \forall i = 1,2,..,I \qquad (15)$$

In the above formulation, zlm_j is a binary variable that will be 1 if $xlm_j > 0$ and 0 otherwise. Similarly zmh_j is a binary variable that will be 1 if $Xmh_j > 0$ and 0 otherwise. Xlm_j denotes the amount of product produced from a production level between l_j and m_j whereas xmh_j denotes the amount of product produced from a production level between m_j and h_j. It can be thus seen that a product can be simultaneously produced from all the possible levels. Equation (4) ensures that Xlm_j is either 0 or between l_j and

m_j where the production and investment cost values can be determined while Equation (5) ensures $Xmh_j > 0$ that is either 0 or is between m_j and h_j. As shown in Equation (10), X_j denotes the total amount of product that is produced from the jth process. $PClm_j$ and $IClm_j$ indicate the production cost and investment cost required for producing Xlm_j and are given by Equation (8) and Equation (9) respectively. Similarly, $PCmh_j$ and $ICmh_j$ indicate the production and investment cost required to produce Xmh_j and are given by Equation (6) and Equation (7) respectively. The total production and investment cost of the jth process are given by PC_j and IC_j respectively and are determined by Equation (11) and Equation (12) respectively. Equation (14) ensures that the total investment cost is below the investment budget whereas Equation (13) ensures that the raw materials required for the production of is not greater than the amount of the particular raw material that is available. Equation (15) is used to ensure that the total production of the ith product from all the processes is less than the demand of the ith product. This constraint is implemented as in literature and can be easily modified according to the specific needs of the user. For example, if there is a penalty incurred if a demand is not satisfied, it may be more appropriate to ensure that the production is more than the demand. The domain of Xlm_j is restricted to integers in $(0, m_j)$ whereas the domain of Xmh_j is restricted to integers in $(0, h_j)$. In case of unique process requirement, the following two constraints are added to the above CP model

$$Z_j = \min\left(1, X_j\right) \qquad \forall j = 1,2,...,J \qquad (16)$$

$$\sum_{j=1}^{J} P_{ij} Z_j \leq 1 \qquad \forall i = 1,2,..,I \qquad (17)$$

Thus the proposed model involves $3J$ integer decision variables corresponding to Xlm_j, Xmh_j and X_j and $3J$ binary decision variables corresponding to zlm_j, zmh_j and Z_j. It can be seen that the above formulation is easy to build and understand. It uses the Boolean 'or' constraints denoted by the symbol ‖ and the Boolean 'and' constraints denoted by the symbol &. Such constraints can easily be accommodated due to the highly expressive modeling power of CP and are available in many of the CP based solvers. It is easy to note that the above formulation accurately determines the production cost and investment cost between the interval (l_j, m_j) and (m_j, h_j) using the corresponding line that governs it. Moreover in this proposed formulation, if profitable, the jth process can produce a product quantity greater than h_j, due to the incorporation of Xlm_j and Xmh_j. It should be noted that, if desired, the above special constraints could be possibly converted using discrete variables into the conventional inequalities of the form $\leq$, $=$ and $\geq$ to obtain a MILP formulation. However such a strategy including its computational complexity is beyond the scope of this work. Though the above formulation is postulated for the

production planning in a petrochemical industry, it can also be used in the production planning of any multi-level production processes. Also, it can also be realized that the above formulation can be easily extend to accommodate more than three levels of production capacity.

V. RESULTS AND DISCUSSIONS

In this section, we demonstrate the benefits of the proposed CP formulation on two scenarios with each involving four different instances. Instance 1 to Instance 4 comprises Scenario I and it enforces the unique process requirement whereas Instance 5 to 8 comprises Scenario II and does not enforce the unique process requirement. The four instances in each case study differ in the amount of investment budget that is available for the investment. Both the raw materials in all that the eight instances are assumed to be 50 units.

TABLE III: OPTIMAL PROFIT AND RESOURCE UTILIZATION

		Item	CP	MILP	% Profit Increase
Scenario I (Unique Process Requirement)	1	B ($)	500	500	0.58
		R_1 (units/yr)	0	0	
		R_2 (units /yr)	0	48	
		Profit ($/yr)	642	638.3	
	2	B ($)	650	650	2.38
		R_1 (units /yr)	0	0	
		R_2 (units /yr)	0	48.2	
		Profit ($/yr)	859	839	
	3	B ($)	750	750	0.81
		R_1 (units /yr)	0	0	
		R_2 (units /yr)	0	48	
		Profit ($/yr)	990	982	
	4	B ($)	798	800	3.68
		R_1 (units/yr)	0	0	
		R_2 (units/yr)	0	48	
		Profit ($/yr)	1070	1032	
Scenario II (No Unique Process Requirement)	5	B ($)	500	500	0.58
		R_1 (units/yr)	0	0	
		R_2 (units/yr)	0	48	
		Profit ($/yr)	642	638.3	
	6	B ($)	650	650	2.38
		R_1 (units/yr)	0	0	
		R_2 (units/yr)	0	48	
		Profit ($/yr)	859	839	
	7	B ($)	750	750	0.81
		R_1 (units/yr)	0	0	
		R_2 (units/yr)	0	48	
		Profit ($/yr)	990	982	
	8	B ($)	798	800	3.68
		R_1 (units/yr)	0	0	
		R_2 (units/yr)	0	48	
		Profit ($/yr)	1070	1032	

The results reported in this section were obtained using the default settings of IBM ILOG CPLEX Optimization Studio on a Processor Intel i5-2400 CPU @ 3.1 GHz. IBM ILOG CPLEX Optimization Studio allows the use of decision variables and decision expressions. To better utilize the computational resources, Equation (4)-(5), (13)-(17) are treated as constraints and are included in the "subject to" part of the model while Equation (2)-(3), (6)-(12) are considered as decision expressions.

Table III details the comparison of profit obtained using CP and the MILP formulation. It also provides the amount of raw materials and the amount of investment budget that is utilized in the corresponding production plan. In all the eight

instances, it can be seen that CP is able to provide a better objective than the MILP formulation in literature. The increase in profit percentage ranges from 0.58 to 3.68. This increment in profit is due to the fact that the CP formulation permits higher production of a profitable product whereas the MILP formulation restricts the production of a profitable product to be less than or equal to h_j. This can be seen from the production plan provided in Table IV for all the eight instances. From Table IV, for Instance 1, it can be seen that the Process 36 is selected to produce 139 units of Product Q using both xlm_j and xmh_j production levels. However from Table I, it can be seen that the value of h_j for Process 36 is 86 and thus the MILP formulation cannot produce more than 86 units. This leads to suboptimal solution being provided by the MILP formulation in literature.

TABLE IV: PRODUCTION PLAN FOR OPTIMAL PROFIT

		Proposed CP Formulation					MILP Formulation		
		N_T	N_S	xlm_j	xmh_j	X_j	N_T	N_S	X_j
Scenario I (Unique Process Requirement)	1	Q	36	58	81	139	H	18	48
							Q	36	35
							V	50	53.7
	2	Q	36	58	60	118	H	18	48.16
		V	50	0	65	65	Q	36	76.34
							V	50	56
	3	Q	36	58	58	116	H	18	48
		V	50	38	56	94	Q	36	61
							S	42	44
							V	50	56
	4	Q	36	56	58	114	H	18	48
		V	50	56	56	112	Q	36	73.5
							S	42	44
							V	50	56
Scenario II (No Unique Process Requirement)	5	Q	36	58	81	139	H	18	48
							Q	36	35
							V	50	53.7
	6	Q	36	58	61	119	H	18	48
		V	50	0	64	64	Q	36	58
							V	50	74.5
	7	Q	36	52	58	110	H	18	48
		S	42	0	44	44	Q	36	61
		V	50	0	56	56	S	42	44
							V	50	56
	8	Q	36	58	58	116	H	18	48
		V	50	54	56	110	Q	36	73.5
							S	42	44
							V	50	56

From Table IV for Instance 1, it can be seen that there are two units of Process 36 which are used to produce 139 units of Product Q since a single unit of h_j cannot produce more than 86. The complete budget of $500 in this case is utilized and hence there is neither additional production of Product Q nor the production of any other product. For all the eight instances in Table IV, it can be observed that Product Q is produced from Process 36. A careful analysis of the data in Table I and Table II shows that the profit per unit of Product Q is higher and it does not depend on either raw material R_1 or R_2 which are available in limited quantities. Thus the only restriction on the amount of Product Q that can be produced is due to the investment budget. The MILP formulation restricts the

amount of Product Q that can be produced to h_j whereas the CP formulation restricts the amount of Product Q that can be produced to $\left(m_j + h_j \right)$ which leads to higher profit.
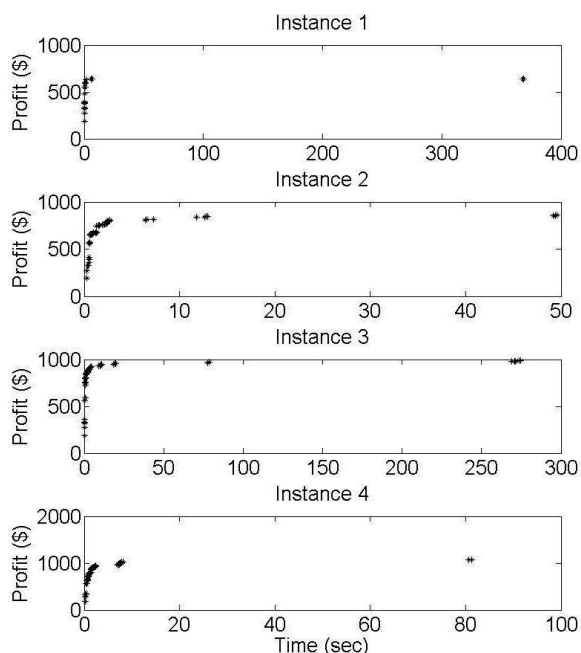

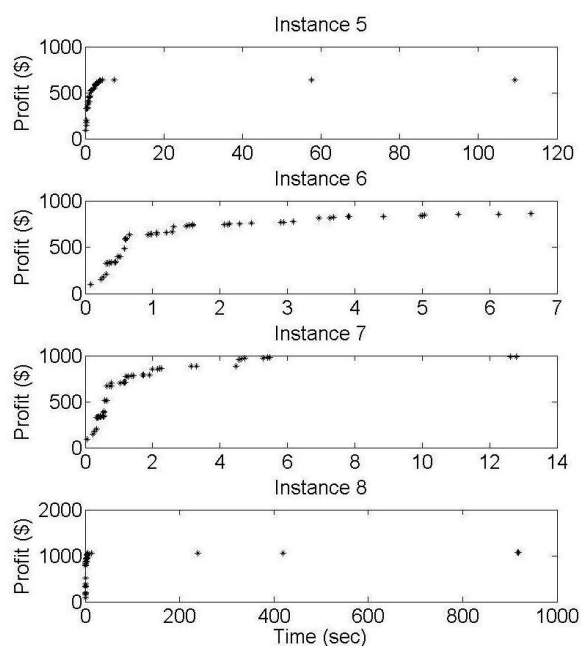Fig. 3. Computational performance of CP for scenario I.

Fig. 4. Computational performance of CP for scenario II.

It can be seen that the profit in Instance 1 and Instance 5 provided by the CP formulation are identical and the production plans are also identical. However, for Instance 2 and Instance 6, the profit provided by CP is identical but the production plans are not identical. Thus, these two production plans can be considered as realizations or multiple solutions as they have identical objective function value but have different sets of decision variables. Similarly, the CP solutions given in Instance 3 and Instance 7 are realizations similar to those between Instance 4 and Instance 8. However, the MILP solutions in Instance 4 and Instance 8 are identical solutions similar to the pair of solutions in Instance 3 and

Instance 7. The computational performance of all the eight instances is shown in Fig. 3 and Fig. 4. It can be seen that CP is able to quickly determine the feasible solutions. The values reported in Table II correspond to the solutions determined by CP after 3600 seconds. It was observed that these solutions were obtained much earlier but there is no further improvement in the solutions provided by CP. As mentioned earlier, this is a drawback of CP that it is computationally intensive, particularly to ensure global optimality. However, it should be noted that this is a design problem and it is not prohibitive to spend some computational effort to determine the profitable solutions. Moreover as the objective of this work is to provide an alternate formulation overcoming the drawbacks of the formulation in literature, additional techniques to direct the search and significantly improve the computational time are not explored in this article and will be a subject of future work.

VI. CONCLUSIONS

In this article, we have proposed a constraint programming based model for determining the optimal production planning in a petrochemical industry. The proposed formulation overcomes the limitations of the MILP formulation in the literature and leads to better solutions. Future work can include incorporation of appropriate cuts and developing a hybrid strategy to reduce the computational efforts required by the proposed model. Additionally, other objectives that affect the petrochemical industries can be accommodated to determine efficient production plans in the midst of conflicting objectives.

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